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GENERALIZED NETS WITH CHARACTERISTICS OF THE ARCS

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Abstract

A new extension of the class of the ordinary generalzed nets is defined – Generalized Nets with Characteristics of the Arcs (GNCA). In it, the arcs can obtain characteristics during the functioning of the net if tokens have passed through them. It is proved that the class of all GNCA is conservative extension of the class of the ordinary generalized nets.

Key words: generalized nets, generalized nets extensions, generalized nets with characteristics of the arcs

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1. Introduction. Many extensions of the Generalized Nets (GNs) have been defined and their properties have been studied [1,3,4]. Among them, the most recent ones are Generalized nets with volumetric tokens (see [6]) and Generalized Nets with Characteristics of the Places (GNCP; see [2]). All of them have been proven to be conservative extensions of the class of the ordinary GNs.

In the ordinary GNs the results of the functioning of the net are preserved as characteristics of the tokens. These characteristics serve as the memory of the net and they are related not only to the tokens that carry them during their stay in the net but, also, to the place where they are obtained by the tokens. However, if we want to collect information related to a specific place, we have to use additional place and tokens which may significantly increase the complexity of the graphical representation of the net. This leads to the definition of the GNCP in which the places can also receive characteristics.

As in the case with the places, if we would like to preserve all information related to a specific arc in a transition of a GN, we need to use additional places and tokens. To avoid this, here we propose a new extension of the class of the ordinary GNs in which the arcs can also receive characteristics. We shall refer to it as Generalized Nets with Characteristics of the Arcs (GNCA).

2. Definition of GNCA. Every transition of a GNCA has the same components as the transitions of the ordinary GNs, i.e. it is described by a seven-tuple:

$$Z = \langle L', L'', t_1, t_2, r, M, \square \rangle,$$

where:

(a) L' and L'' are finite non-empty sets of places (the transition's input and output places, respectively);

(b) t_1 is the current time-moment of the transition's firing;

(c) t_2 is the current value of the duration of its active state;

(d) r is the transition's *condition* determining which tokens will be transferred from the transition's inputs to its outputs. Parameter r has the form of an Index Matrix (IM; see [5]);

(e) M is an IM of the capacities of the transition's arcs;

(f) $\square$ is called transition type and it is an object having a form similar to a Boolean expression. It may contain as variables the symbols that serve as labels for transition's input places and the Boolean connectives $\wedge$ and $\vee$.

Generalized Net with Characteristics of the Arcs (GNCA) is the ordered four-tuple:

$$E = \langle \langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2 \rangle, \langle K, \pi_K, \theta_K \rangle, \langle T, t^0, t^* \rangle, \langle X, \Omega, \Phi, \Xi, b \rangle \rangle,$$

where

(a) A is the set of transitions of the net;

(b) π_A is a function giving the priorities of the transitions, i.e. $\pi_A : A \to \mathcal{N}$, where $\mathcal{N} = \{0, 1, 2, ...\}$;

(c) π_L is a function giving the priorities of the places, i.e. $\pi_L : L \to \mathcal{N}$, where L is the set of all GN places;

(d) c is a function giving the capacities of the places, i.e. $c : L \to \mathcal{N}$;

(e) f is a function that calculates the truth values of the predicates of the transition's conditions;

(f) function

$$\theta_1 = \bigcup_{Z \in A} \theta_1^Z$$

and θ_1^Z is a function giving the next moment of time in which transition Z can be activated;

(g) function

$$\theta_2 = \bigcup_{Z \in A} \theta_2^Z$$

and θ_2^Z is a function giving the duration of the active state of a given transition Z;

(h) K is the set of the GNCA's tokens;

(i) π_K is a function giving the priorities of the tokens, i.e., $\pi_K : K \to \mathcal{N}$;

(j) θ_K is a function giving the time moment when a given token can enter the net, i.e. $\theta_K(\alpha) = t$, where $\alpha \in K$ and $t \in [T, T + t^*]$;

(k) T is the moment of time when the GNCA starts functioning. This moment is determined with respect to a fixed (global) time scale;

(l) t^0 is an elementary time step related to the fixed (global) time scale;

(m) t^* is the duration of the GNCA functioning;

(n) X is a function which assigns initial characteristics to every token when it enters input place of the net;

(o) Ω is a function which assigns initial characteristics to the arcs (this is a new component which cannot be found in the definition of the ordinary GN);

(p) Φ is a characteristic function that assigns new characteristics to every token when it makes a transfer from an input to an output place of a given transition;

(q) Ξ is a characteristic function that assigns new characteristics to the arcs when tokens have passed through them (this is the second new component which cannot be found in the definition of the ordinary GN);

(r) b is a function giving the maximum number of characteristics a given token can receive, i.e., $b : K \to \mathcal{N}$.

3. GNCA are conservative extenstions of the ordinary GNs. Let Σ be the class of all GNs and Σ_{CA} be the class of all GNCA. Every ordinary GN is a GNCA, i.e. the relation

$$\Sigma_{CA} \vdash \Sigma \tag{1}$$

holds (see [3]). This is true because every GN can be viewed as a GNCA in which no arcs obtain characteristics, i.e. $\Omega_{l_i l_j} = \varnothing$ and $\Xi_{l_i l_j} = \varnothing$ for all arcs. Thus, GNCA are extensions of the ordinary GNs. More interesting is that every GNCA can be represented by an ordinary GN.

Theorem 1. *The class Σ_{CA} is conservative extension of the class Σ, i.e.*

$$\Sigma_{CP} \equiv \Sigma\,. \tag{2}$$

Proof. To prove (2) it is enough to show that

$$\Sigma \vdash \Sigma_{CA} \tag{3}$$

In the proof we use the theorem for the completeness of the GN transition [3].

Let E be an arbitrary GNCA:

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2\rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Omega, \Phi, \Xi, b\rangle\rangle\,.$$

We shall construct an ordinary GN which describes the functioning and preserves the results of work of E. For every transition

$$Z = \langle L', L'', t_1, t_2, r, M, \Box\rangle$$

of E we construct a corresponding transition

$$Z^* = \langle L'^*, L''^*, t_1^*, t_2^*, r^*, M^*, \Box^* \rangle .$$

The new transition Z^* has one additional place l_Z which is both input and output for the transition. In place l_Z a token α_Z will be used to preserve the characteristics that the arcs of transition Z obtain in E. The initial characteristic of α_Z is a list of the arcs and their initial characteristics. Place l_Z has the lowest priority among the places of the transition Z^*. For the components of Z^* we have $L'^* = L' \cup \{l_Z\}\,, L''^* = L'' \cup \{l_Z\}\,, t_1^* = t_1\,, t_2^* = t_2\,, \Box^* = \wedge(\Box, l_Z)\,.$

If $r = pr_5 Z = [L', L'', \{r_{l_i,l_j}\}]$ is the IM of the transition's condition, then

$$r^* = pr_5 Z^* = [L' \cup \{l_Z\}, L'' \cup \{l_Z\}, \{r^*_{l_i,l_j}\}],$$

where

$$(\forall l_i \in L')(\forall l_j \in L'')(r^*_{l_i,l_j} = r_{l_i,l_j}),$$

$$(\forall l_i \in L')(\forall l_j \in L'')(r^*_{l_i,l_Z} = r^*_{l_Z,l_j} = \text{“}false\text{”}),$$

$$r^*_{l_Z,l_Z} = \text{“}true\text{”}.$$

If

$$M = pr_6 Z = [L', L'', \{m_{l_i,l_j}\}]$$

is the IM of the capacities of the arcs, then

$$M^* = pr_6 Z^* = [L' \cup \{l_Z\}, L'' \cup \{l_Z\}, \{m^*_{l_i,l_j}\}],$$

where

$$(\forall l_i \in L')(\forall l_j \in L'')(m^*_{l_i,l_j} = m_{l_i,l_j}),$$

$$(\forall l_i \in L')(\forall l_j \in L'')(m^*_{l_i,l_Z} = m^*_{l_Z,l_j} = 0),$$

$$m^*_{l_Z,l_Z} = 1.$$

Let us denote by A^* the set of all transitions obtained from the transitions of E in the same way as we constructed Z^* above. Let E^* be a standard GN

$$E^* = \langle\langle A^*, \pi_A, \pi_L^*, c^*, f, \theta_1, \theta_2\rangle, \langle K^*, \pi_K^*, \theta_K^*\rangle, \langle T, t^0, t^*\rangle, \langle X^*, \Phi^*, b^*\rangle\rangle,$$

where

$$\pi_L^* = \pi_L \cup \pi_{\{l_Z | Z \in A\}}.$$

Function $\pi_{\{l_Z|Z\in A\}}$ determines the priorities of the new places that are elements of the set $\{l_Z | Z \in A\}$ and the priorities of the places l_Z for every transition $Z^* \in A^*$ are the minimal among the priorities of all places of transition Z. $c^* = c \cup c_{\{l_Z|Z\in A\}}$, where function $c_{\{l_Z|Z\in A\}}$ determines the capacities of the additional places l_Z and $c_{l_Z} = 1$ for every place l_Z.

$$K^* = (\bigcup_{l \in Q^I} K_l) \bigcup \{\alpha_Z | Z \in A\},$$

where Q^I is the set of the input places of the net.

$$\theta_K^* = \theta_K \cup \theta_{\{l_Z | Z \in A\}},$$

where function $\theta_{\{l_Z|Z\in A\}}$ determines that each of the α_Z tokens stays in the initial time moment in the corresponding place l_Z.

$$X^* = X \cup \{x_0^{\alpha_Z} | Z \in A\},$$

where $x_0^{\alpha_Z}$ is the initial characteristic of token α_Z and that is a list of all arcs of Z and their initial characteristics:

$$\text{“}\langle l_1' l_1'', \Omega_{l_1' l_1''}\rangle, \langle l_1' l_2'', \Omega_{l_1' l_2''}\rangle, \cdots, \langle l_m' l_n'', \Omega_{l_m' l_n''}\rangle\text{”}.$$

$$\Phi^* = \Phi \cup \Phi^{**}_{\{l_Z | Z \in A\}},$$

where function $\Phi^{**}_{\{l_Z|Z\in A\}}$ determines the characteristics obtained by the α_Z-tokens in the l_Z-places for every transition $Z^* \in A^*$ and this is a list of all arcs with the characteristics obtained by them in the form:

$$\Phi^{**}_{l_Z} = \text{“}\langle\langle l_1' l_1'', \Xi_{l_1' l_1''}\rangle, \langle l_1' l_1'', \Xi_{l_1' l_2''}\rangle, \cdots, \langle l_m' l_n'', \Xi_{l_m' l_n''}\rangle\rangle\text{”}.$$

Finally,

$$b^* = b \cup b_{\{\alpha_z | Z \in A\}},$$

where function $b_{\{\alpha_Z|Z\in A\}}$ determines the number of characteristics that the α_Z-tokens can keep and for every token α_Z we have $b_{\alpha_Z} = \infty$.

We shall prove that the so constructed GN E^* describes the functioning and preserves the results of work of the GNCA E.

Let Z and Z^* be two corresponding transitions of E and E^*. Let us consider two corresponding tokens α and α^* with the same characteristics which are in two corresponding input places of Z and Z^*. If the transfer of α to any output place of Z is impossible, the same will be true for α^* since the predicates of the transitions' condition are the same in both transitions. If token α is transferred to some output place of Z, then token α^* will be transferred to the corresponding output place of Z^*. This is true because at least at the initial moment of time all conditions for transfer (capacities of the arcs, capacities of output places, etc.) are the same in any two corresponding transitions and by induction it will be true at any consecutive moment. The two tokens receive the same characteristics in the corresponding output places. If the arc between the input and output place in Z receives characteristic through the function Ξ, the same characteristic is assigned to the token α_Z in place l_Z of Z^*. The initial characteristic of the arc is initial characteristic of token α_Z. Therefore, E^* represents the functioning and the results of work of the GNCA E.

This completes the proof of (3). From (1) and (3) we obtain (2). $\square$

4. Conlusions and future work. The GNCA defined in this paper can be used to model transport networks where roads, railway lines or inland waterways can be represented in the model by arcs. In such models the possibility to assign characteristics to the arcs significantly simplifies the graphical representation of the net and, also, makes the model closer to the real system.

We proved that $\Sigma_{CA} \equiv \Sigma$. The same has been proven for the GNCP in [2]. Therefore, every GNCA can be represented by a GNCP and every GNCP can be represented by a GNCA. However, a direct constructive proof for this would be useful as it would show how we can construct a net from one of the two types given a net from the other. It is also interesting to define and study an extension of the GNs in which tokens, arcs and places can receive characteristics. In our future work, we shall study various classes of reduced GNCA. It is already known that there exist two classes of minimal reduced GNs: one in which only the tokens receive characteristics, and one in which only the places receive characteristics. The GNCA allow us to define and study a third class of minimal reduced GNs in which only the arcs receive characteristics.

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